Spatial Reasoning

Surface Area and Volume

1. $L = Ph$
$= 20(10) = 200 \text{ cm}^2$

$S = L + 2B$
$= 200 + 2(5)(5)$
$= 250 \text{ cm}^2$

2. $L = Ph$
$= 6(12)(15)$
$= 1080 \text{ m}^2$

apothem of base is $6\sqrt{3}$ m
$S = L + 2B$
$= 1080 + 2\left(\frac{1}{2}(6\sqrt{3})(72)\right)$
$= 1080 + 432\sqrt{3}$
$\approx 1828.2 \text{ m}^2$

3. $L = Ph$
$= 3(8)(14)$
$= 336 \text{ ft}^2$

altitude of base is $4\sqrt{3}$ ft
$S = L + 2B$
$= 336 + 2\left(\frac{1}{2}(8)(4\sqrt{3})\right)$
$= 336 + 32\sqrt{3}$
$\approx 391.4 \text{ ft}^2$

4. $L = 2\pi rh = 2\pi(5.5)(7) = 77\pi \text{ in}^2$
$S = 2\pi rh + 2\pi r^2 = 77\pi + 2\pi(5.5)^2 = 137.5\pi \text{ in}^2$

5. $L = 2\pi rh = 2\pi(4)(23) = 184\pi \text{ cm}^2$
$S = 2\pi rh + 2\pi r^2 = 184\pi + 2\pi(4)^2 = 216\pi \text{ cm}^2$

6. **Step 1** Use the base circumference to find the radius.
$C = 2\pi r$
$16\pi = 2\pi r$
$r = 8 \text{ yd}$
Step 2 Use the radius to find the lateral area and the base area. The height is 3 times the radius, or 24 yd.
$L = 2\pi rh = 2\pi(8)(24) = 384\pi \text{ yd}^2$
$S = 2\pi rh + 2\pi r^2 = 384\pi + 2\pi(8)^2 = 512\pi \text{ yd}^2$

7. The base of the right triangular prism is a rt. △, because 6, 8, 10 is a Pythagorean triple.
The surface area of the right triangular prism is
$S = Ph + 2B$
$= 24(9) + 2\left(\frac{1}{2}(6)(8)\right) = 264 \text{ cm}^2$.
A right cylinder is removed from the triangular prism.
The lateral area of cylinder is
$L = 2\pi rh$
$= 2\pi(2)(9) = 36\pi \text{ cm}^2$.
The base area of cylinder is
$B = \pi r^2 = \pi(2)^2 = 4\pi \text{ cm}^2$.
The surface area of the composite figure is the sum of areas of all surfaces on the exterior of the figure.
$S =$ (prism surface area) + (cylinder lateral area) − (cylinder base area)
$= 264 + 36\pi - 2(4\pi)$
$= 264 + 28\pi \approx 352.0 \text{ cm}^2$

8. The surface area of right rectangular prism is
$S = Ph + 2B$
$= 8(0.5) + 2(2)(2) = 12 \text{ ft}^2$.
A right cylinder is added to the rectangular prism.
The lateral area of cylinder is
$L = 2\pi rh$
$= 2\pi(0.5)(2) = 2\pi \text{ ft}^2$.
The base area of cylinder is not added, just lowered through 2 ft.
The surface area of the composite figure is the sum of the areas of all surfaces on the exterior of figure.
$S =$ (prism surface area) + (cylinder lateral area)
$= 12 + 2\pi \approx 18.3 \text{ ft}^2$

9. original:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(4.5)(11) + 2\pi(4.5)^2$
$= 139.5\pi \text{ ft}^2$
dimensions tripled:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(13.5)(33) + 2\pi(13.5)^2$
$= 1255.5\pi \text{ ft}^2$
$1255.5\pi = 9(139.5\pi)$. So, surface area is multiplied by 9.

10. original:
$S = Ph + 2B$
$= 42(3) + 2(12)(9) = 342 \text{ ft}^2$
dimensions doubled:
$S = Ph + 2B$
$= 84(6) + 2(24)(18) = 1368 \text{ ft}^2$
$1368 = 4(342)$. So, surface area is multiplied by 4.

11. left cell:
$S = Ph + 2B$
$= 90(7) + 2(35)(10) = 1330\ \mu m^2$
right cell:
$S = Ph + 2B$
$= 52(15) + 2(15)(11) = 1110\ \mu m^2$
The cell that measures 35 μm by 7 μm by 10 μm should absorb at a greater rate.

12. **Step 1** Find the base perimeter and the area.
The base perimeter is 4(6) = 24 ft.
The base area is $(6)^2 = 36$ ft^2.
Step 2 Find the lateral area.
The slant height is $\sqrt{3^2 + 4^2} = 5$ ft.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(24)(5) = 60$ ft^2
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 60 + 36 = 96$ ft^2

13. **Step 1** Find the base perimeter and the altitude.
The base perimeter is 3(40) = 120 cm.
The altitude of base is $20\sqrt{3}$ ft. So, the base area is $\frac{1}{2}bh = \frac{1}{2}(40)\left(20\sqrt{3}\right) = 400\sqrt{3}$ cm^2.
Step 2 Find the lateral area.
The slant height is $\sqrt{25^2 - 20^2} = 15$ cm.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(120)(15) = 900$ cm^2
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 900 + 400\sqrt{3} \approx 1592.8$ cm^2

14. **Step 1** Find the base perimeter and the apothem.
The base perimeter is 6(7) = 42 ft.
The apothem is $3.5\sqrt{3}$ ft. So, the base area is $\frac{1}{2}aP = \frac{1}{2}\left(3.5\sqrt{3}\right)(42) = 73.5\sqrt{3}$ ft^2.
Step 2 Find the lateral area.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(42)(15) = 315$ ft^2
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 315 + 73.5\sqrt{3} \approx 442.3$ ft^2

15. $L = \pi r\ell = \pi(11.5)(23) = 264.5\pi$ cm^2
$S = \pi r\ell + \pi r^2 = 264.5\pi + \pi(11.5)^2 = 396.75\pi$ cm^2

16. $\ell = \sqrt{12^2 + 35^2} = 37$ in.
$L = \pi r\ell = \pi(12)(37) = 444\pi$ in^2
$S = \pi r\ell + \pi r^2 = 444\pi + \pi(12)^2 = 588\pi$ in^2

17. $h = 2(8) - 1 = 15$ m; $\ell = \sqrt{8^2 + 15^2} = 17$ m
$L = \pi r\ell = \pi(8)(17) = 136\pi$ m^2
$S = \pi r\ell + \pi r^2 = 136\pi + \pi(8)^2 = 200\pi$ m^2

18. original:
$S = \frac{1}{2}P\ell + \frac{1}{2}aP$
$= \frac{1}{2}(24)(12) + \frac{1}{2}\left(2\sqrt{3}\right)(24)$
$= \left(144 + 24\sqrt{3}\right)$ ft^2
dimensions divided by 3:
$S = \frac{1}{2}P\ell + \frac{1}{2}aP$
$= \frac{1}{2}(8)(4) + \frac{1}{2}\left(\frac{2\sqrt{3}}{3}\right)(8)$
$= \left(16 + \frac{8\sqrt{3}}{3}\right)$ ft^2
$\left(16 + \frac{8\sqrt{3}}{3}\right) = \left(144 + 24\sqrt{3}\right) \div 9$. So, the surface area is divided by 9.

19. original:
$S = \pi r\ell + \pi r^2 = \pi(2)(5) + \pi(2)^2 = 14\pi$ m^2
dimensions doubled:
$S = \pi r\ell + \pi r^2 = \pi(4)(10) + \pi(4)^2 = 56\pi$ m^2
$56\pi = 4(14\pi)$. So, the surface area is multiplied by 4.

20. lateral area of left cone $= \pi(7)(24) = 168\pi$ in^2
lateral area of right cone $= \pi(7)(17) = 119\pi$ in^2
$S =$ (lateral area of left cone)
+ (lateral area of right cone)
$= 168\pi + 119\pi = 287\pi$ in^2

Spatial Reasoning

Surface Area and Volume

21. lateral area of left pyramid
$= \frac{1}{2}P\ell = \frac{1}{2}(36)(15) = 270 \text{ cm}^2$
lateral area of cube $= Ph = (36)(9) = 324 \text{ cm}^2$
lateral area of right pyramid
$= \frac{1}{2}P\ell = \frac{1}{2}(36)(19) = 342 \text{ cm}^2$
$S =$ (lateral area of left pyramid)
$+$ (lateral area of cube)
$+$ (lateral area of right pyramid)
$= 270 + 324 + 342 = 936 \text{ cm}^2$

22. $S = \pi r\ell = \frac{1}{2}\pi\ell^2$
$\pi r(6) = \frac{1}{2}\pi(6)^2$
$r = 3$
$d = 2(3) = 6$ in.

23. $V = Bh = \left(\frac{1}{2}(9)(15)\right)(12) = 810 \text{ yd}^3$

24. $V = Bh$
$= \frac{1}{2}aP \cdot h$
$= \frac{1}{2}\left(\frac{5}{\tan 36°}\right)(50)(15) \approx 2580.7 \text{ m}^3$

25. $B = s^2$
$49 = s^2$
$s = 7$ ft
$h = 7 - 2 = 5$ ft
$V = Bh = (49)(5) = 245 \text{ ft}^3$

26. **Step 1** Find the volume in cubic feet.
$V = \ell wh = (9)(16)\left(\frac{1}{3}\right) = 48 \text{ ft}^3$
Step 2 Use the conversion factor $\frac{1 \text{ yd}^3}{27 \text{ ft}^3}$ to find the volume in cubic yards. Then round up to nearest cubic yard.
$V = 48 \cdot \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 1.78 \text{ yd}^3$
Colin must buy 2 yd^3 of dirt.
Step 3 Use the cost per yard to find the cost of dirt.
Cost = 2($25) = $50

27. $V = \pi r^2 h$
$= \pi(14)^2(9) = 1764\pi \text{ cm}^3$
$\approx 5541.8 \text{ cm}^3$

28. $V = \pi r^2 h$
$= \pi(6)^2(3) = 108\pi \text{ in}^3$
$\approx 339.3 \text{ in}^3$

29. $V = Bh$
$= 24\pi(16) = 384\pi \text{ cm}^3$
$\approx 1206.4 \text{ cm}^3$

30. original dimensions:
$V = \pi r^2 h = \pi(2)^2(3) = 12\pi \text{ yd}^3$
dimensions multiplied by 5:
$V = \pi r^2 h = \pi(10)^2(15) = 1500\pi \text{ yd}^3$
$1500\pi = 125(12\pi)$. So, the volume is multiplied by 125.

31. original dimensions:
$V = Bh = (5)^2(10) = 250\pi \text{ m}^3$
dimensions multiplied by $\frac{3}{5}$:
$V = \pi r^2 h = \pi(3)^2(6) = 54\pi \text{ m}^3$
$54\pi = \frac{27}{125}(250\pi)$. So, the volume is multiplied by $\frac{27}{125}$.

32. volume of lower cube: $V = s^3 = (8)^3 = 512 \text{ cm}^3$
volume of middle cube: $V = s^3 = (6)^3 = 216 \text{ cm}^3$
volume of upper cube: $V = s^3 = (4)^3 = 64 \text{ cm}^3$
total volume: $V = 512 + 216 + 64 = 792 \text{ cm}^3$

33. volume of square-based prism:
$V = Bh = (4)^2(12) = 192 \text{ ft}^3$
volume of each half-cylinder:
$V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(2)^2(4) = 8\pi \text{ ft}^3$
total volume:
$V = 8\pi + 192 + 8\pi = 192 + 16\pi \approx 242.3 \text{ ft}^3$

34. **Step 1** Find the area of the base.
$B = \ell w$
$= (8)(6) = 48 \text{ ft}^2$
Step 2 Use the base and height to find the volume.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(48)(10) = 160 \text{ ft}^3$

35. **Step 1** Find the area of the base. $5^2 + 12^2 = 13^2$. So, the base is a right triangle with $b = 12$ m and h = 5 m.
$B = \frac{1}{2}bh$
$= \frac{1}{2}(12)(5) = 30 \text{ m}^2$
Step 2 Use the base and height to find the volume.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(30)(9) = 90 \text{ m}^2$

36. **Step 1** Find the height.
$6^2 + h^2 = 10^2$
$h = 8$ ft
Step 2 Use the height and base edge length to find the volume.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(12^2)(8) = 384 \text{ ft}^3$

37. **Step 1** Find the volume in cubic feet. The height is 5(3) = 15 ft.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(45^2)(15) = 10{,}125 \text{ ft}^3$
Step 2 Convert the volume to cubic yards.
Use the conversion factor $\frac{1 \text{ yd}^3}{27 \text{ ft}^3}$.
$10{,}125 \text{ ft}^3 \cdot \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 375 \text{ yd}^3$

38. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(9)^2(41) = 1107\pi \text{ m}^3$
$\approx 3477.7 \text{ m}^3$

39. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(2)^2(4) = \frac{16}{3}\pi \text{ in}^3$
$\approx 16.8 \text{ in}^3$

40. $B = \pi r^2$
$36\pi = \pi r^2$
$r = 6$ ft
$h = 2r = 2(6) = 12$ ft
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(36\pi)(12) = 144\pi \text{ ft}^3$
$\approx 452.4 \text{ ft}^3$

41. original dimensions:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(15)^2(21) = 1575\pi \text{ in}^3$
dimensions multiplied by $\frac{1}{3}$:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(5)^2(7) = \frac{175}{3}\pi \text{ in}^3$
Notice that $\frac{175}{3}\pi = \frac{1}{27}(1575\pi)$. If the dimensions are multiplied by $\frac{1}{3}$, the volume is multiplied by $\frac{1}{27}$.

42. original dimensions:
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(7^2)(4) = \frac{196}{3} \text{ ft}^3$
dimensions multiplied by 6:
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(42^2)(24) = 14{,}112 \text{ cm}^3$
Notice that $14{,}112 = 216\left(\frac{196}{3}\right)$. If the dimensions are multiplied by 6, the volume is multiplied by 216.

43. The volume of the cylinder is
$V = \pi r^2 h = \pi(6)^2(10) = 360\pi \text{ ft}^3$.
The volume of the cone is
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(10) = 120\pi \text{ ft}^3$.
The volume of the composite figure is the difference of the volumes.
$V = 360\pi - 120\pi = 240\pi \text{ ft}^3$
$\approx 754.0 \text{ ft}^3$

44. The volume of the rectangular prism is
$V = \ell wh = (10)(5)(2) = 100 \text{ ft}^3$.
The volume of each square-based pyramid is
$V = \frac{1}{3}Bh = \frac{1}{3}(5^2)(3) = 25 \text{ ft}^3$.
The volume of the composite figure is the sum of the volumes.
$V = 100 + 25 + 25 = 150 \text{ ft}^3$

45. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 = 972\pi \text{ cm}^3$

46. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(7)^3 = \frac{686}{3}\pi \text{ ft}^3$

47. $V = \frac{4}{3}\pi r^3$
$7776\pi = \frac{4}{3}\pi r^3$
$5832 = r^3$
$r = 18$
$d = 36$ in.

48. 9-mm pearl:
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.5)^3 = 121.5\pi \text{ mm}^3$
6-mm pearl:
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 108\pi \text{ mm}^3$
The 9-mm pearl is 3.375 times as great in volume.

49. $S = 4\pi r^2 = 4\pi(21)^2 = 1764\pi \text{ in}^2$

50. $A = \pi r^2$
$S = 4\pi r^2 = 4A = 4(81\pi) = 324\pi \text{ in}^2$

51. $S = 4\pi r^2$
$625\pi = 4\pi r^2$
$156.25 = r^2$
$r = 12.5$ m
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.5)^3 = \frac{15{,}625}{6}\pi \text{ m}^3$

52. original dimensions:
$S = 4\pi r^2$
$= 4\pi(0.6)^2$
$= 1.44\pi \text{ ft}^2$

dimensions multiplied by $\frac{1}{5}$:
$S = 4\pi r^2$
$= 4\pi(0.12)^2$
$= 0.0576\pi \text{ in.}^2$

Notice that $0.0576\pi = \frac{1}{25}(1.44\pi)$. If the dimensions are multiplied by $\frac{1}{5}$, the surface area is multiplied by $\frac{1}{25}$.

53. original dimensions:
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(14)^3$
$= \frac{10{,}976}{3}\pi \text{ mm}^3$

dimensions multiplied by 6:
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(84)^3$
$= 790{,}272\pi \text{ mm}^3$

Notice that $790{,}272\pi = 216\left(\frac{10{,}976}{3}\pi\right)$. If the radius is multiplied by 6, the volume is multiplied by 216.

54. Step 1 Find the surface area.
$S_{prism} = Ph + 2\ell w$
$= 2(10) + 2(5))(4) + 2(10)(5) = 220 \text{ cm}^2$
$L_{hemisphere} = \frac{1}{2}(4\pi r^2) = 2\pi(3)^2 = 18\pi \text{ cm}^2$
$B_{hemisphere} = \pi r^2 = \pi(3)^2 = 9\pi$
$S = S_{prism} + L_{hemisphere} - B_{hemisphere}$
$= 220 + 18\pi - 9\pi$
$= 220 + 9\pi \approx 248.3 \text{ cm}^2$

Step 2 Find the volume.
$V_{prism} = \ell wh = (10)(5)(4) = 200 \text{ cm}^3$
$V_{hemisphere} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(3)^3 = 18\pi \text{ cm}^3$
$V = V_{prism} + V_{hemisphere} = 200 + 18\pi \approx 256.5 \text{ cm}^3$

55. Step 1 Find the surface area. The slant height of the cone is $\sqrt{20^2 + 24^2} = 26$ mm.
$S_{cone} = \pi r\ell + \pi r^2$
$= \pi(10)(26) + \pi(10)^2 = 360\pi \text{ mm}^2$
$L_{hemisphere} = \frac{1}{2}(4\pi r^2) = 2\pi(8)^2 = 128\pi \text{ mm}^2$
$B_{hemisphere} = \pi r^2 = \pi(8)^2 = 64\pi$
$S = S_{prism} + L_{hemisphere} - B_{hemisphere}$
$= 360\pi + 128\pi - 64\pi$
$= 424\pi \approx 1332.0 \text{ mm}^2$

Step 2 Find the volume.
$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}(10)^2(24) = 800\pi \text{ mm}^3$
$V_{hemisphere} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(8)^3 = \frac{1024}{3}\pi \text{ mm}^3$
$V = V_{cone} - V_{hemisphere}$
$= 800\pi - \frac{1024}{3}\pi = \frac{1376}{3}\pi \approx 1440.9 \text{ mm}^3$