

Spatial Reasoning
Surface Area and Volume

$$\begin{aligned} 1. L &= Ph \\ &= 20(10) = 200 \text{ cm}^2 \end{aligned} \quad \begin{aligned} S &= L + 2B \\ &= 200 + 2(5)(5) \\ &= 250 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 2. L &= Ph \\ &= 6(12)(15) \\ &= 1080 \text{ m}^2 \end{aligned} \quad \begin{aligned} &\text{apothem of base is } 6\sqrt{3} \text{ m} \\ S &= L + 2B \\ &= 1080 + 2\left(\frac{1}{2}(6\sqrt{3})(72)\right) \\ &= 1080 + 432\sqrt{3} \\ &\approx 1828.2 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} 3. L &= Ph \\ &= 3(8)(14) \\ &= 336 \text{ ft}^2 \end{aligned} \quad \begin{aligned} &\text{altitude of base is } 4\sqrt{3} \text{ ft} \\ S &= L + 2B \\ &= 336 + 2\left(\frac{1}{2}(8)(4\sqrt{3})\right) \\ &= 336 + 32\sqrt{3} \\ &\approx 391.4 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} 4. L &= 2\pi rh = 2\pi(5.5)(7) = 77\pi \text{ in}^2 \\ S &= 2\pi rh + 2\pi r^2 = 77\pi + 2\pi(5.5)^2 = 137.5\pi \text{ in}^2 \end{aligned}$$

$$\begin{aligned} 5. L &= 2\pi rh = 2\pi(4)(23) = 184\pi \text{ cm}^2 \\ S &= 2\pi rh + 2\pi r^2 = 184\pi + 2\pi(4)^2 = 216\pi \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 6. \text{ Step 1} &\text{ Use the base circumference to find the radius.} \\ C &= 2\pi r \\ 16\pi &= 2\pi r \\ r &= 8 \text{ yd} \end{aligned}$$

$$\begin{aligned} \text{Step 2} &\text{ Use the radius to find the lateral area and the} \\ &\text{base area. The height is 3 times the radius, or 24 yd.} \\ L &= 2\pi rh = 2\pi(8)(24) = 384\pi \text{ yd}^2 \\ S &= 2\pi rh + 2\pi r^2 = 384\pi + 2\pi(8)^2 = 512\pi \text{ yd}^2 \end{aligned}$$

7. The base of the right triangular prism is a rt. Δ , because 6, 8, 10 is a Pythagorean triple.
The surface area of the right triangular prism is

$$\begin{aligned} S &= Ph + 2B \\ &= 24(9) + 2\left(\frac{1}{2}(6)(8)\right) = 264 \text{ cm}^2. \end{aligned}$$

A right cylinder is removed from the triangular prism.
The lateral area of cylinder is

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(2)(9) = 36\pi \text{ cm}^2. \end{aligned}$$

The base area of cylinder is

$$B = \pi r^2 = \pi(2)^2 = 4\pi \text{ cm}^2.$$

The surface area of the composite figure is the sum of areas of all surfaces on the exterior of the figure.

$$\begin{aligned} S &= (\text{prism surface area}) + (\text{cylinder lateral area}) \\ &\quad - (\text{cylinder base area}) \\ &= 264 + 36\pi - 2(4\pi) \\ &= 264 + 28\pi \approx 352.0 \text{ cm}^2 \end{aligned}$$

8. The surface area of right rectangular prism is

$$\begin{aligned} S &= Ph + 2B \\ &= 8(0.5) + 2(2)(2) = 12 \text{ ft}^2. \end{aligned}$$

A right cylinder is added to the rectangular prism.
The lateral area of cylinder is

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(0.5)(2) = 2\pi \text{ ft}^2. \end{aligned}$$

The base area of cylinder is not added, just lowered through 2 ft.
The surface area of the composite figure is the sum of the areas of all surfaces on the exterior of figure.

$$\begin{aligned} S &= (\text{prism surface area}) + (\text{cylinder lateral area}) \\ &= 12 + 2\pi \approx 18.3 \text{ ft}^2 \end{aligned}$$

9. original:

$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(4.5)(11) + 2\pi(4.5)^2 \\ &= 139.5\pi \text{ ft}^2 \end{aligned}$$

dimensions tripled:

$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(13.5)(33) + 2\pi(13.5)^2 \\ &= 1255.5\pi \text{ ft}^2 \end{aligned}$$

$1255.5\pi = 9(139.5\pi)$. So, surface area is multiplied by 9.

10. original:

$$\begin{aligned} S &= Ph + 2B \\ &= 42(3) + 2(12)(9) = 342 \text{ ft}^2 \end{aligned}$$

dimensions doubled:

$$\begin{aligned} S &= Ph + 2B \\ &= 84(6) + 2(24)(18) = 1368 \text{ ft}^2 \end{aligned}$$

$1368 = 4(342)$. So, surface area is multiplied by 4.

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11. left cell:

$$S = Ph + 2B$$

$$= 90(7) + 2(35)(10) = 1330 \mu\text{m}^2$$

right cell:

$$S = Ph + 2B$$

$$= 52(15) + 2(15)(11) = 1110 \mu\text{m}^2$$

The cell that measures $35 \mu\text{m}$ by $7 \mu\text{m}$ by $10 \mu\text{m}$ should absorb at a greater rate.

 13. **Step 1** Find the base perimeter and the altitude.

The base perimeter is $3(40) = 120 \text{ cm}$.
 The altitude of base is $20\sqrt{3} \text{ ft}$. So, the base area is $\frac{1}{2}bh = \frac{1}{2}(40)(20\sqrt{3}) = 400\sqrt{3} \text{ cm}^2$.

Step 2 Find the lateral area.

The slant height is $\sqrt{25^2 - 20^2} = 15 \text{ cm}$.

$$L = \frac{1}{2}P\ell$$

$$= \frac{1}{2}(120)(15) = 900 \text{ cm}^2$$

Step 3 Find the surface area.

$$S = \frac{1}{2}P\ell + B$$

$$= 900 + 400\sqrt{3} \approx 1592.8 \text{ cm}^2$$

$$15. L = \pi r \ell = \pi(11.5)(23) = 264.5\pi \text{ cm}^2$$

$$S = \pi r \ell + \pi r^2 = 264.5\pi + \pi(11.5)^2 = 396.75\pi \text{ cm}^2$$

$$17. h = 2(8) - 1 = 15 \text{ m}; \ell = \sqrt{8^2 + 15^2} = 17 \text{ m}$$

$$L = \pi r \ell = \pi(8)(17) = 136\pi \text{ m}^2$$

$$S = \pi r \ell + \pi r^2 = 136\pi + \pi(8)^2 = 200\pi \text{ m}^2$$

19. original:

$$S = \pi r \ell + \pi r^2 = \pi(2)(5) + \pi(2)^2 = 14\pi \text{ m}^2$$

dimensions doubled:

$$S = \pi r \ell + \pi r^2 = \pi(4)(10) + \pi(4)^2 = 56\pi \text{ m}^2$$

$$56\pi = 4(14\pi). \text{ So, the surface area is multiplied by 4.}$$

 12. **Step 1** Find the base perimeter and the area.

The base perimeter is $4(6) = 24 \text{ ft}$.

The base area is $(6)^2 = 36 \text{ ft}^2$.

Step 2 Find the lateral area.

The slant height is $\sqrt{3^2 + 4^2} = 5 \text{ ft}$.

$$L = \frac{1}{2}P\ell$$

$$= \frac{1}{2}(24)(5) = 60 \text{ ft}^2$$

Step 3 Find the surface area.

$$S = \frac{1}{2}P\ell + B$$

$$= 60 + 36 = 96 \text{ ft}^2$$

 14. **Step 1** Find the base perimeter and the apothem.

The base perimeter is $6(7) = 42 \text{ ft}$.

The apothem is $3.5\sqrt{3} \text{ ft}$. So, the base area is

$$\frac{1}{2}aP = \frac{1}{2}(3.5\sqrt{3})(42) = 73.5\sqrt{3} \text{ ft}^2.$$

Step 2 Find the lateral area.

$$L = \frac{1}{2}P\ell$$

$$= \frac{1}{2}(42)(15) = 315 \text{ ft}^2$$

Step 3 Find the surface area.

$$S = \frac{1}{2}P\ell + B$$

$$= 315 + 73.5\sqrt{3} \approx 442.3 \text{ ft}^2$$

$$16. \ell = \sqrt{12^2 + 35^2} = 37 \text{ in.}$$

$$L = \pi r \ell = \pi(12)(37) = 444\pi \text{ in}^2$$

$$S = \pi r \ell + \pi r^2 = 444\pi + \pi(12)^2 = 588\pi \text{ in}^2$$

18. original:

$$S = \frac{1}{2}P\ell + \frac{1}{2}aP$$

$$= \frac{1}{2}(24)(12) + \frac{1}{2}(2\sqrt{3})(24)$$

$$= (144 + 24\sqrt{3}) \text{ ft}^2$$

dimensions divided by 3:

$$S = \frac{1}{2}P\ell + \frac{1}{2}aP$$

$$= \frac{1}{2}(8)(4) + \frac{1}{2}\left(\frac{2\sqrt{3}}{3}\right)(8)$$

$$= \left(16 + \frac{8\sqrt{3}}{3}\right) \text{ ft}^2$$

$\left(16 + \frac{8\sqrt{3}}{3}\right) = (144 + 24\sqrt{3}) \div 9$. So, the surface area is divided by 9.

$$20. \text{ lateral area of left cone} = \pi(7)(24) = 168\pi \text{ in}^2$$

$$\text{ lateral area of right cone} = \pi(7)(17) = 119\pi \text{ in}^2$$

$$S = (\text{lateral area of left cone})$$

$$+ (\text{lateral area of right cone})$$

$$= 168\pi + 119\pi = 287\pi \text{ in}^2$$

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21. lateral area of left pyramid
 $= \frac{1}{2}P\ell = \frac{1}{2}(36)(15) = 270 \text{ cm}^2$
 lateral area of cube $= Ph = (36)(9) = 324 \text{ cm}^2$
 lateral area of right pyramid
 $= \frac{1}{2}P\ell = \frac{1}{2}(36)(19) = 342 \text{ cm}^2$
 $S = (\text{lateral area of left pyramid})$
 $\quad + (\text{lateral area of cube})$
 $\quad + (\text{lateral area of right pyramid})$
 $= 270 + 324 + 342 = 936 \text{ cm}^2$
22. $S = \pi r \ell = \frac{1}{2}\pi \ell^2$
 $\pi r(6) = \frac{1}{2}\pi(6)^2$
 $r = 3$
 $d = 2(3) = 6 \text{ in.}$
23. $V = Bh = \left(\frac{1}{2}(9)(15)\right)(12) = 810 \text{ yd}^3$
24. $V = Bh$
 $= \frac{1}{2}aP \cdot h$
 $= \frac{1}{2}\left(\frac{5}{\tan 36^\circ}\right)(50)(15) \approx 2580.7 \text{ m}^3$
25. $B = s^2$
 $49 = s^2$
 $s = 7 \text{ ft}$
 $h = 7 - 2 = 5 \text{ ft}$
 $V = Bh = (49)(5) = 245 \text{ ft}^3$
26. **Step 1** Find the volume in cubic feet.
 $V = \ell wh = (9)(16)\left(\frac{1}{3}\right) = 48 \text{ ft}^3$
Step 2 Use the conversion factor $\frac{1 \text{ yd}^3}{27 \text{ ft}^3}$ to find the volume in cubic yards. Then round up to nearest cubic yard.
 $V = 48 \cdot \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 1.78 \text{ yd}^3$
 Colin must buy 2 yd^3 of dirt.
Step 3 Use the cost per yard to find the cost of dirt.
 Cost $= 2(\$25) = \50
27. $V = \pi r^2 h$
 $= \pi(14)^2(9) = 1764\pi \text{ cm}^3$
 $\approx 5541.8 \text{ cm}^3$
28. $V = \pi r^2 h$
 $= \pi(6)^2(3) = 108\pi \text{ in}^3$
 $\approx 339.3 \text{ in}^3$
29. $V = Bh$
 $= 24\pi(16) = 384\pi \text{ cm}^3$
 $\approx 1206.4 \text{ cm}^3$
30. original dimensions:
 $V = \pi r^2 h = \pi(2)^2(3) = 12\pi \text{ yd}^3$
 dimensions multiplied by 5:
 $V = \pi r^2 h = \pi(10)^2(15) = 1500\pi \text{ yd}^3$
 $1500\pi = 125(12\pi)$. So, the volume is multiplied by 125.
31. original dimensions:
 $V = Bh = (5)^2(10) = 250\pi \text{ m}^3$
 dimensions multiplied by $\frac{3}{5}$:
 $V = \pi r^2 h = \pi(3)^2(6) = 54\pi \text{ m}^3$
 $54\pi = \frac{27}{125}(250\pi)$. So, the volume is multiplied by $\frac{27}{125}$.
32. volume of lower cube: $V = s^3 = (8)^3 = 512 \text{ cm}^3$
 volume of middle cube: $V = s^3 = (6)^3 = 216 \text{ cm}^3$
 volume of upper cube: $V = s^3 = (4)^3 = 64 \text{ cm}^3$
 total volume: $V = 512 + 216 + 64 = 792 \text{ cm}^3$
33. volume of square-based prism:
 $V = Bh = (4)^2(12) = 192 \text{ ft}^3$
 volume of each half-cylinder:
 $V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(2)^2(4) = 8\pi \text{ ft}^3$
 total volume:
 $V = 8\pi + 192 + 8\pi = 192 + 16\pi \approx 242.3 \text{ ft}^3$
34. **Step 1** Find the area of the base.
 $B = \ell w$
 $= (8)(6) = 48 \text{ ft}^2$
Step 2 Use the base and height to find the volume.
 $V = \frac{1}{3}Bh$
 $= \frac{1}{3}(48)(10) = 160 \text{ ft}^3$

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35. **Step 1** Find the area of the base. $5^2 + 12^2 = 13^2$.
So, the base is a right triangle with $b = 12$ m and $h = 5$ m.
 $B = \frac{1}{2}bh$
 $= \frac{1}{2}(12)(5) = 30 \text{ m}^2$
Step 2 Use the base and height to find the volume.
 $V = \frac{1}{3}Bh$
 $= \frac{1}{3}(30)(9) = 90 \text{ m}^2$
36. **Step 1** Find the height.
 $6^2 + h^2 = 10^2$
 $h = 8$ ft
Step 2 Use the height and base edge length to find the volume.
 $V = \frac{1}{3}Bh$
 $= \frac{1}{3}(12^2)(8) = 384 \text{ ft}^3$
37. **Step 1** Find the volume in cubic feet. The height is $5(3) = 15$ ft.
 $V = \frac{1}{3}Bh$
 $= \frac{1}{3}(45^2)(15) = 10,125 \text{ ft}^3$
Step 2 Convert the volume to cubic yards.
Use the conversion factor $\frac{1 \text{ yd}^3}{27 \text{ ft}^3}$.
 $10,125 \text{ ft}^3 \cdot \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 375 \text{ yd}^3$
38. $V = \frac{1}{3}\pi r^2 h$
 $= \frac{1}{3}\pi(9)^2(41) = 1107\pi \text{ m}^3$
 $\approx 3477.7 \text{ m}^3$
39. $V = \frac{1}{3}\pi r^2 h$
 $= \frac{1}{3}\pi(2)^2(4) = \frac{16}{3}\pi \text{ in}^3$
 $\approx 16.8 \text{ in}^3$
40. $B = \pi r^2$
 $36\pi = \pi r^2$
 $r = 6$ ft
 $h = 2r = 2(6) = 12$ ft
 $V = \frac{1}{3}Bh$
 $= \frac{1}{3}(36\pi)(12) = 144\pi \text{ ft}^3$
 $\approx 452.4 \text{ ft}^3$
41. original dimensions:
 $V = \frac{1}{3}\pi r^2 h$
 $= \frac{1}{3}\pi(15)^2(21) = 1575\pi \text{ in}^3$
dimensions multiplied by $\frac{1}{3}$:
 $V = \frac{1}{3}\pi r^2 h$
 $= \frac{1}{3}\pi(5)^2(7) = \frac{175}{3}\pi \text{ in}^3$
Notice that $\frac{175}{3}\pi = \frac{1}{27}(1575\pi)$. If the dimensions are multiplied by $\frac{1}{3}$, the volume is multiplied by $\frac{1}{27}$.
42. original dimensions:
 $V = \frac{1}{3}Bh$
 $= \frac{1}{3}(7^2)(4) = \frac{196}{3} \text{ ft}^3$
dimensions multiplied by 6:
 $V = \frac{1}{3}Bh$
 $= \frac{1}{3}(42^2)(24) = 14,112 \text{ cm}^3$
Notice that $14,112 = 216\left(\frac{196}{3}\right)$. If the dimensions are multiplied by 6, the volume is multiplied by 216.
43. The volume of the cylinder is
 $V = \pi r^2 h = \pi(6)^2(10) = 360\pi \text{ ft}^3$.
The volume of the cone is
 $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(10) = 120\pi \text{ ft}^3$.
The volume of the composite figure is the difference of the volumes.
 $V = 360\pi - 120\pi = 240\pi \text{ ft}^3$
 $\approx 754.0 \text{ ft}^3$
44. The volume of the rectangular prism is
 $V = \ell wh = (10)(5)(2) = 100 \text{ ft}^3$.
The volume of each square-based pyramid is
 $V = \frac{1}{3}Bh = \frac{1}{3}(5^2)(3) = 25 \text{ ft}^3$.
The volume of the composite figure is the sum of the volumes.
 $V = 100 + 25 + 25 = 150 \text{ ft}^3$
45. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 = 972\pi \text{ cm}^3$
46. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(7)^3 = \frac{686}{3}\pi \text{ ft}^3$

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$$\begin{aligned}
 47. \quad V &= \frac{4}{3}\pi r^3 \\
 7776\pi &= \frac{4}{3}\pi r^3 \\
 5832 &= r^3 \\
 r &= 18 \\
 d &= 36 \text{ in.}
 \end{aligned}$$

$$49. S = 4\pi r^2 = 4\pi(21)^2 = 1764\pi \text{ in}^2$$

$$\begin{aligned}
 51. \quad S &= 4\pi r^2 \\
 625\pi &= 4\pi r^2 \\
 156.25 &= r^2 \\
 r &= 12.5 \text{ m} \\
 V &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.5)^3 = \frac{15,625}{6}\pi \text{ m}^3
 \end{aligned}$$

$$\begin{array}{ll}
 53. \text{ original dimensions:} & \text{dimensions multiplied by 6:} \\
 V = \frac{4}{3}\pi r^3 & V = \frac{4}{3}\pi r^3 \\
 = \frac{4}{3}\pi(14)^3 & = \frac{4}{3}\pi(84)^3 \\
 = \frac{10,976}{3}\pi \text{ mm}^3 & = 790,272\pi \text{ mm}^3
 \end{array}$$

Notice that $790,272\pi = 216\left(\frac{10,976}{3}\pi\right)$. If the radius is multiplied by 6, the volume is multiplied by 216.

$$\begin{aligned}
 54. \text{ Step 1 Find the surface area.} \\
 S_{\text{prism}} &= Ph + 2\ell w \\
 &= 2(10) + 2(5)(4) + 2(10)(5) = 220 \text{ cm}^2 \\
 L_{\text{hemisphere}} &= \frac{1}{2}(4\pi r^2) = 2\pi(3)^2 = 18\pi \text{ cm}^2 \\
 B_{\text{hemisphere}} &= \pi r^2 = \pi(3)^2 = 9\pi \\
 S &= S_{\text{prism}} + L_{\text{hemisphere}} - B_{\text{hemisphere}} \\
 &= 220 + 18\pi - 9\pi \\
 &= 220 + 9\pi \approx 248.3 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 55. \text{ Step 1 Find the surface area. The slant height of} \\
 \text{the cone is } \sqrt{20^2 + 24^2} &= 26 \text{ mm.} \\
 S_{\text{cone}} &= \pi r\ell + \pi r^2 \\
 &= \pi(10)(26) + \pi(10)^2 = 360\pi \text{ mm}^2 \\
 L_{\text{hemisphere}} &= \frac{1}{2}(4\pi r^2) = 2\pi(8)^2 = 128\pi \text{ mm}^2 \\
 B_{\text{hemisphere}} &= \pi r^2 = \pi(8)^2 = 64\pi \\
 S &= S_{\text{prism}} + L_{\text{hemisphere}} - B_{\text{hemisphere}} \\
 &= 360\pi + 128\pi - 64\pi \\
 &= 424\pi \approx 1332.0 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 48. \text{ 9-mm pearl:} \\
 V &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.5)^3 = 121.5\pi \text{ mm}^3 \\
 \text{6-mm pearl:} \\
 V &= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 108\pi \text{ mm}^3 \\
 \text{The 9-mm pearl is 3.375 times as great in volume.}
 \end{aligned}$$

$$\begin{aligned}
 50. \quad A &= \pi r^2 \\
 S &= 4\pi r^2 = 4A = 4(81\pi) = 324\pi \text{ in}^2
 \end{aligned}$$

$$\begin{array}{ll}
 52. \text{ original dimensions:} & \text{dimensions multiplied by } \frac{1}{5}: \\
 S = 4\pi r^2 & S = 4\pi r^2 \\
 = 4\pi(0.6)^2 & = 4\pi(0.12)^2 \\
 = 1.44\pi \text{ ft}^2 & = 0.0576\pi \text{ in}^2
 \end{array}$$

Notice that $0.0576\pi = \frac{1}{25}(1.44\pi)$. If the dimensions are multiplied by $\frac{1}{5}$, the surface area is multiplied by $\frac{1}{25}$.

$$\begin{aligned}
 \text{Step 2 Find the volume.} \\
 V_{\text{prism}} &= \ell wh = (10)(5)(4) = 200 \text{ cm}^3 \\
 V_{\text{hemisphere}} &= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(3)^3 = 18\pi \text{ cm}^3 \\
 V &= V_{\text{prism}} + V_{\text{hemisphere}} = 200 + 18\pi \approx 256.5 \text{ cm}^3
 \end{aligned}$$

$$\begin{aligned}
 \text{Step 2 Find the volume.} \\
 V_{\text{cone}} &= \frac{1}{3}\pi r^2 h = \frac{1}{3}(10)^2(24) = 800\pi \text{ mm}^3 \\
 V_{\text{hemisphere}} &= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(8)^3 = \frac{1024}{3}\pi \text{ mm}^3 \\
 V &= V_{\text{cone}} - V_{\text{hemisphere}} \\
 &= 800\pi - \frac{1024}{3}\pi = \frac{1376}{3}\pi \approx 1440.9 \text{ mm}^3
 \end{aligned}$$