Thinkwell's Placement Test 6 Answer Key

If you answered 7 or more Test 6 questions correctly, we recommend Thinkwell's Precalculus. If you answered fewer than 7 Test 6 questions correctly, we recommend Thinkwell's Algebra 2.

1. **Answer: –3**

Explanation

The given function $f(x)$ is a piecewise function.

$$f(x)=\begin{cases}-2x \text{ if } x<-2\\ 5x+7 \text{ if } -2\le x\le 4\\ -7x \text{ if } x>4\end{cases}$$

The domain is divided into three parts: x-values less than -2, x-values between -2 and 4 (including -2 and 4), and x-values greater than 4. For all x-values less than -2, the function used is $f(x)=-2x$. For all x-values between -2 and 4, the function used is $f(x)=5x+7$. And, for all values greater than 4, the function used is $f(x)=-7x$. Therefore, the second function, $f(x)=5x+7$, is used to evaluate $f(-2)$, since -2 is in the domain of this piece of the function. $f(-2)=5(-2)+7=-3$

Thus, $f(-2)=-3$.

This concept is covered in Thinkwell's Algebra 2 topic "Piecewise Functions."

2. **Answer: vertex (3, 2); $x = 3$**

Explanation

The vertex of a parabola defined by a quadratic equation, $y=ax^2+bx+c$, is located at the point (h,k), where $h=-\frac{b}{2a}$ and $k=ah^2+bh+c$. So, to find the coordinates of a parabola's vertex, first find the value of h, and then substitute that h-value into the given equation to find the value of k. The given equation, $y=5x^2-30x+47$, is in the standard form of a quadratic equation, $y=ax^2+bx+c$, where $a=5$, $b=-30$, and $c=47$.

Use the values of a and b to find the x-coordinate of the vertex, h.

$$h=-\frac{b}{2a}=-\frac{-30}{2(5)}=3$$

So, the vertex's x-coordinate is 3.

Now substitute 3 into the given equation to find the value of k, which is the vertex's y-coordinate.

$$\begin{aligned} y &= 5x^2-30x+47 \\ &= 5(3)^2-30(3)+47 && \textit{Substitute 3 for x.} \\ &= 5(9)-30(3)+47 && \textit{Evaluate the power.} \\ &= 2 && \textit{Simplify.} \end{aligned}$$

So, the vertex's y-coordinate is 2.

Therefore, the parabola's vertex is at $(3, 2)$.

The axis of symmetry of a vertically opening parabola is a vertical line through the parabola's vertex. Since it is the x-variable that is squared in the given equation, the parabola opens vertically. Therefore, the axis of symmetry is the vertical line that passes through the vertex, $(3, 2)$. The equation of the vertical line that passes through $(3, 2)$ is $x=3$. Thus, the axis of symmetry is the line $x=3$.

This concept is covered in Thinkwell's Algebra 2 topic "Properties of Quadratic Functions in Standard Form."

3. Answer: $x=\frac{3\pm i\sqrt{23}}{4}$

Explanation

Use the quadratic formula to solve the equation. First, set the equation equal to 0.

$$2x^2-3x=-4$$

$$2x^2-3x+4=0 \quad \textit{Add 4 to each side.}$$

Now substitute the values of a, b, and c into the quadratic formula.

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-(-3)\pm\sqrt{(-3)^2-4(2)(4)}}{2(2)} && \textit{Substitute.} \\ &= \frac{3\pm\sqrt{-23}}{4} && \textit{Simplify under the radical.} \\ &= \frac{3\pm i\sqrt{23}}{4} && \textit{Simplify the square root.} \end{aligned}$$

This concept is covered in Thinkwell's Algebra 2 topic "The Quadratic Formula."

4. **Answer: $y = 3$**

Explanation

$$\sqrt{6y} = 3\sqrt{2y-4}$$

$$\left(\sqrt{6y}\right)^2 = \left(3\sqrt{2y-4}\right)^2 \quad \textit{Square each side.}$$

$$\left(\sqrt{6y}\right)^2 = (3)^2\left(\sqrt{2y-4}\right)^2 \quad \textit{Power of a Product Property}$$

$$6y = 9(2y-4) \quad \textit{Evaluate each power.}$$

$$6y = 18y - 36 \quad \textit{Distributive Property}$$

$$-12y = -36 \quad \textit{Subtract 18y from each side.}$$

$$y = 3 \quad \textit{Divide each side by - 12.}$$

Substitute the solution into the original equation to determine if the solution is extraneous.

$$\sqrt{6(3)} = 3\sqrt{2(3)-4} \quad \textit{Substitute.}$$

$$\sqrt{18} = 3\sqrt{6-4} \quad \textit{Multiply.}$$

$$\sqrt{18} = 3\sqrt{2} \quad \textit{Subtract.}$$

$$\sqrt{9\cdot 2} = 3\sqrt{2} \quad \textit{Factor.}$$

$$3\sqrt{2} = 3\sqrt{2} \quad \textit{Product of Square Roots Property}$$

Therefore, the solution is $y = 3$.

This concept is covered in Thinkwell's Algebra 2 topic "Solving Radical Equations and Inequalities."

5. **Answer: $9x^2 - 30x + 25$**

Explanation

The expression within the 2nd set of parentheses contains like terms. So, simplify that expression first, and then multiply by using the Distributive Property. Here, subtraction is written as addition of a negative number in order to keep the signs straight when using the Distributive Property.

$$(x-2)\left(x^2-4x+1+x\right)$$

$$(x-2)\left(x^2-3x+1\right) \quad \textit{Combine like terms.}$$

$$\left(x+(-2)\right)\left(x^2+(-3x)+1\right) \quad \textit{Write subtraction as addition of a negative.}$$

$$x\left(x^2+(-3x)+1\right)+(-2)\left(x^2+(-3x)+1\right) \quad \textit{Distribute.}$$

$$x\left(x^2\right)+x(-3x)+x(1)+(-2)\left(x^2\right)+(-2)(-3x)+(-2)(1) \quad \textit{Distribute.}$$

$$x^3+\left(-3x^2\right)+x+\left(-2x^2\right)+6x+(-2) \quad \textit{Multiply.}$$

$$x^3-3x^2+x-2x^2+6x-2 \quad \textit{Write addition of a negative as subtraction.}$$

$$x^3-5x^2+7x-2 \quad \textit{Combine like terms.}$$

This concept is covered in Thinkwell's Algebra 2 topic "Multiplying Polynomials."

6. **Answer:** $\mathbf{\dfrac{5x-11}{x^2-1}}$

Explanation

Begin by factoring the denominators, if possible. $\dfrac{5}{x+1}-\dfrac{6}{x^2-1}=\dfrac{5}{x+1}-\dfrac{6}{(x+1)(x-1)}$

Before the rational expressions can be subtracted, they must first be written as equivalent rational expressions with a common denominator. So, find the common denominator. Since the denominator of the first rational expression is $x+1$ and the denominator of the second rational expression is $(x+1)(x-1)$, the common denominator is also $(x+1)(x-1)$. Since the denominator of the second rational expression is the common denominator, that expression does not need to be rewritten.

The first rational expression needs to be written as an equivalent rational expression where the denominator is $(x+1)(x-1)$.

So, multiply the first rational expression by $\dfrac{x-1}{x-1}$ and then subtract.

$$\frac{5}{x+1}-\frac{6}{(x+1)(x-1)}$$

$$\frac{5}{x+1}\cdot\frac{x-1}{x-1}-\frac{6}{(x+1)(x-1)}$$ *Multiply the first rational expression by x − 1 over itself.*

$$\frac{5(x-1)}{(x+1)(x-1)}-\frac{6}{(x+1)(x-1)}$$ *Multiply the numerators and multiply the denominators.*

$$\frac{5(x-1)-6}{(x+1)(x-1)}$$ *Subtract the 2nd numerator from the 1st numerator.*

$$\frac{5x-5-6}{x^2-1}$$ *Distribute 5 in the numerator. FOIL the denominator.*

$$\frac{5x-11}{x^2-1}$$ *Combine like terms in the numerator.*

This concept is covered in Thinkwell's Algebra 2 topic "Adding and Subtracting Rational Expressions."

7. **Answer: $\mathbf{-2x+1}$, R(2)**

Explanation

Since the denominator does not include an x-term, add the placeholder $0x$ to the denominator. Use long division to divide.

$$\frac{6x^3-3x^2-2x+3}{-3x^2+1}=\frac{6x^3-3x^2-2x+3}{-3x^2+0x+1}$$

$$\begin{array}{r} -2x+1 \\ -3x^2+0x+1\overline{\big)\,6x^3-3x^2-2x+3} \\ \underline{-(6x^3+0x^2-2x)} \\ -3x^2+0x+3 \\ \underline{-(-3x^2+0x+1)} \\ 2 \end{array}$$

Thus, the quotient is $-2x+1$, R(2).

This concept is covered in Thinkwell's Algebra 2 topic "Dividing Polynomials."

8. **Answer: $x = -4$ with multiplicity three; $x = 3$ with multiplicity one**

Explanation

Since $x = -4$ is a zero, $x + 4$ is a factor. Use synthetic division to divide the polynomial by the factor $x + 4$.

$$\begin{array}{r|rrrrr} -4 & 1 & 9 & 12 & -80 & -192 \\ & & -4 & -20 & 32 & 192 \\ \hline & 1 & 5 & -8 & -48 & 0 \end{array}$$

So, $\dfrac{x^4 + 9x^3 + 12x^2 - 80x - 192}{x + 4} = x^3 + 5x^2 - 8x - 48.$

It follows that $x^4 + 9x^3 + 12x^2 - 80x - 192 = (x + 4)\left(x^3 + 5x^2 - 8x - 48\right).$

Since $x = 3$ is a zero, $x - 3$ is a factor. Use synthetic division to divide $x^3 + 5x^2 - 8x - 48$ by $x - 3$.

$$\begin{array}{r|rrrr} 3 & 1 & 5 & -8 & -48 \\ & & 3 & 24 & 48 \\ \hline & 1 & 8 & 16 & 0 \end{array}$$

So, $\dfrac{x^3 + 5x^2 - 8x - 48}{x - 3} = x^2 + 8x + 16.$

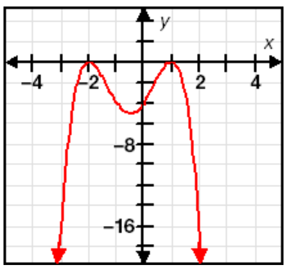
It follows that $x^4 + 9x^3 + 12x^2 - 80x - 192 = (x + 4)\left(x^3 + 5x^2 - 8x - 48\right) = (x + 4)(x - 3)\left(x^2 + 8x + 16\right).$

Factor $x^2 + 8x + 16$. $(x + 4)(x - 3)\left(x^2 + 8x + 16\right) = (x + 4)(x - 3)(x + 4)(x + 4) = (x + 4)^3 (x - 3)$

Thus, the zeros are $x = -4$ with multiplicity three and $x = 3$ with multiplicity one.

This concept is covered in Thinkwell's Algebra 2 topic "Finding Real Roots of Polynomial Equations."

9. **Answer:**



Explanation

Expand the polynomial.

$$\begin{aligned} f(x) &= -(x + 2)^2 (x - 1)^2 \\ &= -\left(x^2 + 4x + 4\right)\left(x^2 - 2x + 1\right) \\ &= -\left[x^2\left(x^2 - 2x + 1\right) + 4x\left(x^2 - 2x + 1\right) + 4\left(x^2 - 2x + 1\right)\right] \\ &= -\left[x^4 - 2x^3 + x^2 + 4x^3 - 8x^2 + 4x + 4x^2 - 8x + 4\right] \\ &= -\left[x^4 + 2x^3 - 3x^2 - 4x + 4\right] \\ &= -x^4 - 2x^3 + 3x^2 + 4x - 4 \end{aligned}$$

Thus, the leading term of $f(x)$ is $-x^4$, which has degree 4 and coefficient of -1.

Therefore, since the degree is even and the coefficient is negative, both ends of the graph point down.

Find the zeros.

$$-(x + 2)^2 (x - 1)^2 = 0$$

$$\begin{array}{ccc} (x + 2)^2 = 0 & \text{or} & (x - 1)^2 = 0 \\ x + 2 = 0 & & x - 1 = 0 \\ x = -2 & & x = 1 \end{array}$$

So, the graph has zeros (x-intercepts) at 1 and -2. Now determine if each x-intercept is a turning point by examining the zero's multiplicity. The multiplicity of both zeros is 2 since the exponent of both related factors is 2. Since the multiplicity is even, the curve has a turning point at both x-intercepts.

Find the y-intercept. $f(0) = -(0 + 2)^2 (0 - 1)^2 = -(2)^2 (-1)^2 = -4$ So, the y-intercept of $f(x)$ is at -4.

Use substitution to find several additional points on the graph near where $x = -2$, $x = 0$, and $x = 1$.

x	$-(x+2)^2(x-1)^2$	$f(x)$	$(x, f(x))$
-3	$-(-3+2)^2(-3-1)^2$	-16	$(-3, -16)$
-1	$-(-1+2)^2(-1-1)^2$	-4	$(-1, -4)$
2	$-(2+2)^2(2-1)^2$	-16	$(2, -16)$

Plot those three points, along with the y-intercept and the x-intercepts. Then, sketch a curve that points down to the far left, goes through $(-3, -16)$, and then turns at $(-2, 0)$ and passes through $(-1, -4)$, and then turns and passes through $(0, -4)$, and continues up and then turns at $(1, 0)$, then continues down to the far right, passing through $(2, -16)$.

This concept is covered in Thinkwell's Algebra 2 topic "Investigating Graphs of Polynomial Functions."

10. **Answer: x = 13**

Explanation

Since one side of the equation is a radical, begin by squaring both sides to remove the radical.

$$\sqrt{4x-3} = x-6$$
$$\left(\sqrt{4x-3}\right)^2 = (x-6)^2$$
$$4x-3 = x^2-12x+36$$

The resulting equation is quadratic. So, set the equation equal to 0 by bringing all terms to either side and then solve by factoring or by using the quadratic formula.

$$4x-3 = x^2-12x+36$$
$$0 = x^2-12x+36-4x+3$$
$$0 = x^2-16x+39$$
$$0 = (x-13)(x-3)$$
$$x-13=0 \quad \text{or} \quad x-3=0$$
$$x=13 \qquad\qquad x=3$$

Check that both solutions satisfy the equation.

$$\boxed{x=13} \qquad\qquad \boxed{x=3}$$
$$\sqrt{4(13)-3} = (13)-6 \qquad \sqrt{4(3)-3} = (3)-6$$
$$\sqrt{49} = 7 \qquad\qquad \sqrt{9} = -3$$
$$7 = 7 \qquad\qquad 3 \neq -3$$

So, $x=3$ is not a solution since it does not satisfy the equation. The solution is $x=13$.

This concept is covered in Thinkwell's Algebra 2 topic "Solving Radical Equations and Inequalities."