Thinkwell's Placement Test 5 Answer Key

If you answered 7 or more Test 5 questions correctly, we recommend Thinkwell's Algebra 2. If you answered fewer than 7 Test 5 questions correctly, we recommend Thinkwell's Algebra 1.

1. **Answer:** $\boldsymbol{x = -\frac{1}{6}}$ **or** $\boldsymbol{x = -\frac{7}{6}}$

Explanation

Set up the two equations, then solve each equation.

$|6x + 4| = 3 \Rightarrow$

$$\begin{aligned} 6x + 4 &= 3 \quad &\text{or} \quad 6x + 4 &= -3 \\ 6x &= -1 & 6x &= -7 \\ x &= -\frac{1}{6} & x &= -\frac{7}{6} \end{aligned}$$

Check.

$x = -\frac{1}{6} \Rightarrow \left|6\left(-\frac{1}{6}\right) + 4\right| = |-1 + 4| = 3$ (checks) $\quad x = -\frac{7}{6} \Rightarrow \left|6\left(-\frac{7}{6}\right) + 4\right| = |-7 + 4| = 3$ (checks)

This concept is covered in Thinkwell's Algebra 1 topic "Solving Absolute-Value Equations."

2. **Answer:** $\boldsymbol{9x^2 - 30x + 25}$

Explanation

$(3x - 5)^2$	
$(3x - 5)(3x - 5)$	*Expand the power.*
$(3x)(3x) + (3x)(-5) + (-5)(3x) + (-5)(-5)$	*FOIL*
$9x^2 + (-15x) + (-15x) + 25$	*Multiply.*
$9x^2 + (-30x) + 25$	*Combine like terms.*
$9x^2 - 30x + 25$	*Write addition of a negative as subtraction.*

This concept is covered in Thinkwell's Algebra 1 topic "Multiplying Binomials."

3. **Answer: 2*s* + 5, R 12**

Explanation

Set up the rational expression as long division where the numerator is the dividend and the denominator is the divisor.

$$\frac{2s^2 - s - 3}{s - 3} = s - 3\overline{)2s^2 - s - 3}$$

Complete the long division.

$$\begin{array}{r} 2s + 5 \\ s - 3\overline{)2s^2 - s - 3} \end{array}$$

$-(2s^2 - 6s)$	*Multiply 2s by s – 3 and subtract.*
$5s - 3$	*Bring down the – 3.*
$-(5s - 15)$	*Multiply 5 by s – 3 and subtract.*
12	*The remainder is 12.*

Thus, the quotient is $2s + 5$, R 12.

This concept is covered in Thinkwell's Algebra 1 topic "Dividing Polynomials."

4. **Answer:** $(3x+2)(5x-6)$

Explanation

The expression $15x^2 - 8x - 12$ is a trinomial. So, if the expression is factorable, it will factor into a product of two binomials where the product of the first terms is $15x^2$ and the product of the last terms is –12. So, if the first term of each binomial is set to be an x-term, then the coefficients of the first terms must be a pair of factors of 15 and the last terms must be a pair of factors of –12.

(□x + □)(□x + □) □ are factors of 15 and □ are factors of –12.

The trinomial's middle term is –8x. Therefore, the sum of the outer product and the inner product must be –8.

Use trial and error to find the factors.

Factors of 15	Factors of –12	Outer + Inner
3·5	3(–4)	3(–4) + 5(3) = 3
3·5	–4(3)	3(3) + 5(–4) = –11
3·5	–3(4)	3(4) + 5(–3) = –3
3·5	4(–3)	3(–3) + 5(4) = 11
3·5	6(–2)	3(–2) + 5(6) = 24
3·5	–2(6)	3(6) + 5(–2) = 8
3·5	2(–6)	3(–6) + 5(2) = –8

Therefore, the coefficients must be 3 and 5 and the constant terms must be 2 and –6. Write the binomials.

$(3x + 2)(5x + -6)$

Write addition of a negative number as subtraction.

$(3x + 2)(5x - 6)$

Check the factors by multiplying. The product of $(3x + 2)$ and $(5x - 6)$ should be $15x^2 - 8x - 12$.

$$(3x+2)(5x-6)$$

$(3x)(5x)+(3x)(-6)+(2)(5x)+(2)(-6)$ *FOIL*

$15x^2+(-18x)+10x+(-12)$ *Multiply.*

$15x^2+(-8x)+(-12)$ *Combine like terms.*

$15x^2-8x-12$ *Write addition of a negative as subtraction.*

This concept is covered in Thinkwell's Algebra 1 topic "Factoring $ax^2 + bx + c$."

5. **Answer:** $5x^2\sqrt{2y}$

Explanation

$$\sqrt{\frac{20x^2y^3z\left(5x^3\right)}{2xy^2z}}$$

$\sqrt{\frac{100x^5y^3z}{2xy^2z}}$ *Multiply 20 and 5. Product of Powers Property :* $x^2 \cdot x^3 = x^{2+3} = x^5$.

$\sqrt{50x^4y}$ *Divide 100 by 2. Quotient of Powers Property :* $\frac{x^5}{x} = x^{5-1} = x^4$, $\frac{y^3}{y^2} = y^{3-2} = y$.

$\sqrt{25\cdot 2\cdot x^4\cdot y}$ *Factor.*

$\sqrt{25x^4}\sqrt{2y}$ *Product Property of Square Roots*

$5x^2\sqrt{2y}$ *Simplify the square roots of perfect squares.*

This concept is covered in Thinkwell's Algebra 1 topic "Radical Expressions."

6. **Answer: $y = -4x - 25$**

The slope, m, and y-intercept, b, of a line must be known in order to write the equation of that line in slope-intercept form, $y = mx + b$. Begin by using the coordinates from the two given points to find the slope of the line. Then use the coordinates from either of the given points and the slope to find the y-intercept. Once the slope and y-intercept are known, substitute those values into slope-intercept form, $y = mx + b$, to write the equation of the line.

Find the slope. Substitute the coordinates from the two points into the slope formula. Let $(-6, -1)$ be (x_1, y_1) and let $(-8, 7)$ be (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-1)}{-8 - (-6)} = \frac{8}{-2} = -4$$

Find the y - intercept. Substitute the slope and the x- and y-coordinates from *either* given point into slope-intercept form and solve for b. Here, the coordinates from $(-6, -1)$ are used to find b.

$y = mx + b$	*Slope - Intercept Form*
$-1 = -4(-6) + b$	*Substitute – 4 for m, – 6 for x, and – 1 for y.*
$-1 = 24 + b$	*Multiply.*
$b = -25$	*Subtract 24 from each side.*

Write the equation. Substitute the m and b values into slope-intercept form, $y = mx + b$. Note that values are *not* substituted in for x and y when the equation of the line is written.

$y = mx + b$	*Slope - Intercept Form*
$y = -4x + (-25)$	*Substitute – 4 for m and – 25 for b.*
$y = -4x - 25$	*Write addition of a negative number as subtraction.*

This concept is covered in Thinkwell's Algebra 1 topic "Point-Slope Form."

7. **Answer: $y = -\frac{3}{4}x + 1$**

Explanation

Find the slope. The slope of $y = \frac{4}{3}x - 1$ is $\frac{4}{3}$. The slopes of perpendicular lines are opposite reciprocals. So, the slope of any line that is perpendicular to $y = \frac{4}{3}x - 1$ is $-\frac{3}{4}$. Thus, $m = -\frac{3}{4}$.

Find the y - intercept. Substitute the slope, $m = -\frac{3}{4}$, and the x- and y-coordinates from the given point, $(4, -2)$, into slope-intercept form and solve for b.

$y = mx + b$	*Slope - Intercept Form*
$-2 = -\frac{3}{4}(4) + b$	*Substitute – 3/4 for m, 4 for x, and – 2 for y.*
$-2 = -3 + b$	*Multiply.*
$b = 1$	*Add 3 to both sides.*

Write the equation. Substitute $m = -3/4$ and $b = 1$ into slope-intercept form.

$y = mx + b$	*Slope - Intercept Form*
$y = -\frac{3}{4}x + 1$	*Substitute – 3/4 for m and 1 for b.*

This concept is covered in Thinkwell's Algebra 1 topic "Slope of Parallel and Perpendicular Lines."

8. **Answer: (4, 3)**

<u>Explanation</u>

The 2nd equation, $x = 19 - 5y$, is solved for x. So, $19 - 5y$ can be substituted into the 1st equation for x.

$$\boxed{x} + 4y = 16$$
$$\nwarrow$$
$$x = \boxed{19 - 5y}$$

After $19 - 5y$ is substituted for x, the resulting equation includes only one variable, y. Simplify the resulting equation and then solve for y.

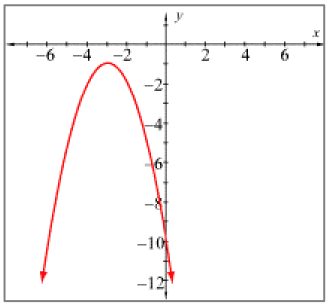
$$
\begin{aligned}
x + 4y &= 16 \\
(19 - 5y) + 4y &= 16 && \textit{Substitute } 19 - 5y \textit{ for } x. \\
19 - y &= 16 && \textit{Combine like terms.} \\
-y &= -3 && \textit{Subtract 19 from each side.} \\
y &= 3 && \textit{Divide each side by } -1.
\end{aligned}
$$

Find the x-coordinate of the solution. Substitute $y = 3$ into *either* of the original equations to find the value of x. Here, 3 will be substituted for y in the equation $x = 19 - 5y$.

$$
\begin{aligned}
x &= 19 - 5y \\
x &= 19 - 5(3) && \textit{Substitute 3 for } y. \\
x &= 19 - 15 && \textit{Multiply.} \\
x &= 4 && \textit{Subtract.}
\end{aligned}
$$

So, the system's solution is $(4,\ 3)$.

This concept is covered in Thinkwell's Algebra 1 topic "Solving Systems by Substitution."

9. **Answer:** 

Explanation

The given function $y = -x^2 - 6x - 10$ is quadratic. Therefore, the shape of the figure is a parabola.

Step 1: Find the axis of symmetry. Use the formula $x = -\frac{b}{2a}$ to find the axis of symmetry. In the given function, *a* = –1 and *b* = –6. Substitute these values into the formula and simplify.

$$x = -\frac{b}{2a} = -\frac{-6}{2(-1)} = -\frac{-6}{-2} = -3$$

Therefore, the parabola's axis of symmetry is the vertical line at *x* = –3.
Step 2: Find the vertex. The vertex is located on the axis of symmetry. Therefore, the *x*-coordinate of the vertex must be *x* = –3. Substitute this *x*-value into the given function to find the *y*-coordinate of the vertex.

$$y = -(-3)^2 - 6(-3) - 10 = -9 - (-18) - 10 = -1$$

Therefore, the parabola's vertex is located at (–3, –1).
Step 3: Find the *y*-intercept. The parabola's *y*-intercept is always located at *c*, so identify *c* from the given function. Since *c* = –10, the parabola's *y*-intercept is located at (0, –10).
Step 4: Find two additional points on the same side of the axis of symmetry as the point containing the *y*-intercept. Since the vertex is at (–3, –1) and the *y*-intercept is at (0, –10), two additional points can be found where *x* = –1 and *x* = –2. Substitute each of these *x*-coordinates into the given function and simplify to find the corresponding *y*-coordinates.

$$x = -1: \quad y = -(-1)^2 - 6(-1) - 10 = -1 - (-6) - 10 = -5$$

$$x = -2: \quad y = -(-2)^2 - 6(-2) - 10 = -4 - (-12) - 10 = -2$$

Therefore, the parabola also passes through the points (–1, –5) and (–2, –2).
Step 5: Graph the axis of symmetry, the vertex, the point containing the *y*-intercept and the two other points.
Step 6: Reflect the point containing the *y*-intercept and the two other points across the axis of symmetry and connect the points with a smooth curve.
This concept is covered in Thinkwell's Algebra 1 topic "Graphing Quadratic Functions."

10. **Answer: E-None of the above**
Explanation
A. The discriminant is the value of $b^2 - 4ac$ where *a*, *b*, and *c* are the coefficients and constant term from a quadratic equation in standard form, $ax^2 + bx + c = 0$. If the discriminant is 0, $b^2 - 4ac = 0$, then the graph of the corresponding function, $y = ax^2 + bx + c$, will be a parabola with exactly one *x*-intercept. If the discriminant is positive, $b^2 - 4ac > 0$, then the graph of the corresponding function, $y = ax^2 + bx + c$, will be a parabola with exactly two *x*-intercepts. If the discriminant is negative, $b^2 - 4ac < 0$, then the graph of the corresponding function, $y = ax^2 + bx + c$, will be a parabola with no *x*-intercepts. The given parabola has exactly one *x*-intercept, the parabola's vertex. Therefore, the discriminant cannot be positive.
B. The constant term of a quadratic function is the parabola's *y*-intercept. So, if the constant term is zero, then the parabola must intersect the *y*-axis at 0. The given parabola's *y*-intercept is at 2, so the constant term is 2.
C. The given parabola opens upwards. Therefore, the coefficient of x^2 must be positive, not negative.
D. The given parabola has exactly one *x*-intercept. Therefore, the discriminant cannot be negative.
This concept is covered in Thinkwell's Algebra 1 topic "The Discriminant."